What you'll Learn About
- How integrate by separating the variables

Find the general solution (Solve for y)

A) $\int \frac{dy}{dx} = \int x+2$

$y = \frac{1}{2}x^2+2x+C$

$\frac{dy}{dx} = x+2$

$\int 1\,dy = \int (x+2)\,dx$

$y = \frac{1}{2}x+2x+C$

B) $(dx)\frac{dy}{dx} = y+2\,(dx)$

$\frac{dy}{y+2} = \frac{(y+2)\,dx}{y+2}$

$\int \frac{1}{y+2}\,dy = \int 1\,dx$

$\ln|y+2| = x+C$

$e^{\ln|y+2|} = e^{x+C}$

$|y+2| = e^{x+C}$

$y = e^{x+C}-2$

$y = e^x(e^C)-2$

$y = Ae^x-2$

Solve the initial value problem - Find C to find the final function

C) $\cancel{(dx)}\frac{dy}{\cancel{dx}} = \frac{5x\,(dx)}{y}$ when $x=1$ and $y=2$

$(y)dy = \frac{5x}{y}\,dx\,(y)$

$\int y\,dy = \int 5x\,dx$

$\frac{1}{2}y^2 = \frac{5}{2}x^2+C$

$\frac{1}{2}(2)^2 = \frac{5}{2}(1)^2+C$

$2 = 2.5+C$

$C = -\frac{1}{2}$

$\frac{1}{2}y^2 = \frac{5}{2}x^2-\frac{1}{2}$

$y^2 = 5x^2-1$

$y = \pm\sqrt{5x^2-1}$

$\frac{1}{2}y^2 = \frac{5}{2}x^2+C$

$y^2 = 5x^2+C$

$y = \sqrt{5x^2+C}$

$2 = \sqrt{5}+C$

$4 = 5+C$

$-1 = C$

$y = \sqrt{5x^2-1}$

1. Get x with dx
   Get y with dy
2. Integrate Both Sides (+C)
3. Plug in values to find C
4. Solve for y

D) $\frac{dy}{dx} = y\sqrt{x}$ when x = 1 and y = 2

$\int \frac{1}{y}\,dy = \int x^{1/2}\,dx$

$\ln|y| = \frac{2}{3}x^{3/2} + C$

$\ln 2 = \frac{2}{3} + C$

$\ln(2) - \frac{2}{3} = C$

$\ln y = \frac{2}{3}x^{3/2} + \ln(2) - \frac{2}{3}$

$e^{\ln y} = e^{\frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}}$

$y = e^{\frac{2}{3}x^{3/2} + \ln 2 - \frac{2}{3}}$

$y = e^{\frac{2}{3}x^{3/2} + C}$

$y = e^{\frac{2}{3}x^{3/2}} e^{C}$

$y = Ce^{\frac{2}{3}x^{3/2}}$

$2 = Ce^{2/3}$

$2/e^{2/3} = C$

$y = \frac{2}{e^{2/3}} e^{\frac{2}{3}x^{3/2}}$

E) $\frac{dy}{dx} = y\sqrt{x}$ when x = 1 and y = −2

$\ln|y| = \frac{2}{3}x^{3/2} + C$

$\ln|-2| = \frac{2}{3} + C$

$\ln 2 = \frac{2}{3} + C$

$\ln(2) - \frac{2}{3} = C$

$\ln(-y) = \frac{2}{3}x^{3/2} + \ln(2) - \frac{2}{3}$

$-y = e^{\frac{2}{3}x^{3/2} + \ln(2) - \frac{2}{3}}$

$y = -e^{\frac{2}{3}x^{3/2} + \ln 2 - 2/3}$

E) $\frac{dy}{dx} = -yx - y \qquad f(-2) = 1$

$\frac{dy}{y} = (-yx - y)dx \rightarrow \frac{dy}{y} = \frac{y(-x-1)}{y}dx$

$\int \frac{1}{y}dy = \int (-x-1)dx$

$\ln|y| = -\frac{1}{2}x^2 - x + C$

$y = e^{-\frac{1}{2}x^2 - x + C} = e^{-\frac{1}{2}x - x} \cdot e^C$

$y = Ae^{-1/2x^2 - x}$

$1 = Ae^0$

$1 = A$

$1 = e^{-2-(-2)+C}$

$1 = e^C$

$C = 1$

$y = 1e^{-\frac{1}{2}x^2 - x}$